

EXAMINATION THE HORIZONTAL MATHEMATIZATION PROCESSES OF SECONDARY SCHOOL STUDENTS ACCORDING TO THE REALISTIC MATHEMATIC EDUCATION: THE EXAMPLE OF PROBABILITY SUBJECT

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Abstract

The purpose of this study is to examine the horizontal mathematization processes of secondary school 8th grade students with an activity on the topic of probability in the theoretical umbrella of Realistic Mathematics Education. This study is qualitative in nature. In order to attract the attention of students, a story was arranged by the researchers. According to the results, the students were generally successful in the horizontal mathematization process. In addition, students appear to be more successful at “solving a problem situated in daily life”, which requires them to make mathematical interpretations; this supports the viewpoint that the success of students increase when relations are established with everyday life situations, as claimed by the reality principle in realistic mathematics teaching.

Key words: Realistic Mathematics, horizontal mathematization, mathematic education.

INTRODUCTION

Realistic Mathematics Education (RME) is a domain-specific instruction theory for mathematics education (Treffers, 1987; De Lange, 1987) The term ‘realistic’ refers more to the aim that students should be presented problem situations which they can *envisage* than that it refers to the ‘realness’ or authenticity of problems. (Wijdeveld, 1980) The reality principle of RME can be acknowledged with two concepts real-life problems and meaningful problem situations. In RME, teaching starts with problems in rich contexts that includes mathematical organization provides informal contextrelated solution strategies as a first step in the learning process rather than beginning with teaching abstractions or formulas. One of the main concepts of RME is Freudenthal’s (1971) idea of mathematics as a human activity. In other words, the students instead of being the receivers of ready-made mathematics, are assumed as active participants in the teaching-learning process that takes place within the social context of the classroom. In RME mathematics was not the body of mathematical knowledge, but the activity of solving problems and looking for problems, and more generally, the activity of organizing matter from reality or mathematical matter. These periods are called ‘mathematization’ (Freudenthal, 1968). Treffers (1978, 1987) who placed the two ways of mathematizing in a new perspective, formulated the idea of two ways of mathematizing in an educational context by distinguishing ‘horizontal’ and ‘vertical’ mathematization.

Many studies have shown that up-to-date, in the case of horizontal mathematization, students can describe a context problem in mathematical terms, organize mathematical tools, discover relations, transfer a real world problem to a mathematical problem, schematize, formulate and visualize a problem in different ways. On the other hand, vertical mathematization is the period of a variety of reorganizations and operations within the mathematical system itself. Through vertical mathematization, the student arrives a higher level of mathematics. If the students encounter a problem they have previously solved at the same level, they employ

horizontal mathematization; if the problem is at a higher further level, they employ vertical mathematization. However, it must be noted that the difference between these two types is not always clear.

Many studies report the contents, purposes and benefits of the RME approach in mathematics education. One of these studies was conducted by Van Den Heuvel-Panhuizen (2003) on the topic of percentages. For example in the study, the sale price and the discount percentage are given and the students have to find out the original price. That the sale price equals 75% of the original price and as an upper level in the study however, the original price can be found by means of a one-step division by dividing the sale price by three fourths or by seventy-five hundredths, which is the opposite of finding the sale price when the original price and the percentage of discount has been given. This solution illustrates an example of vertical mathematizing. In another research, possible effects of RME method for seventh grade students' achievements and attitudes to mathematics was studied. According to the results of the study, the realistic mathematics education method oriented activities has positive effects on the students' achievements (Kaylak, 2014). Similarly, Akkaya (2010) studied about learning environments suitable for RME. Results pointed out that more qualified mathematical knowledge could be created when realistic problems are applied in the education.

Based on the findings of the studies, it may be possible to claim that the RME approach is beneficial for students because it provides rich content, includes everyday life problems and supports formal learning. In studies conducted to date, the effect of RME was examined in different mathematical topics. In the present study, on the other hand, the horizontal mathematization process, which is one of the mathematization processes included in the theoretical umbrella of RME, was examined according to three different themes. These three themes were defined as “describing a context problem in mathematical terms, expressing numbers in different types and solving a problem situated in daily life”. Examining how these three themes were realized in the activity sample as it concerned the probability topic will contribute to the literature on RME. In light of this information, the aim of this study is to examine the horizontal mathematization processes of 8th grade secondary school students with an activity that includes everyday life problems and which is focused on the topic of probability.

METHOD

This study is qualitative in nature and makes use of the case study method. This method examines facts within a specific environment. (Şimşek & Yıldırım, 2006). In this study, the relation between mathematics and everyday life events was defined and themes related to the events were reported. The case study method focused on how the horizontal mathematization processes of students were effected. This study focused on students' horizontal mathematization processes which involved: (1) describing a context problem in mathematical terms; (2) expressing numbers in different types; (3) solving a problem situated within daily life. Qualitative data collection methods such as observation and document analysis were used in this study.

The study was conducted in one sixth-grade classroom in a school based in a large city in Turkey. For this case study, we purposefully selected 35 of 8th grade students who were located in the same class in a secondary school. The participants were labelled using the initials: S1, S2, S3, ..., S37.

To document and analyse the process of developing students' horizontal mathematization processes, “a story that involves subjects in the field of probability in context problems” was arranged by the researchers. This study was designed in a definitive manner to attract the attention of students. In addition, the questions in the study were asked to different student groups as a pre-application; then, questions that were misunderstood were reorganized and the opinions of two lecturers were taken into account during the process of preparing this story in order to provide validity. These problems were designed to attain realistic considerations of the problems involved, in order to derive the correct answers. The problems that formed part of the story, that was wrote by researchers of this research are shown down.

The Story About Probability Subject

Osman, Erdem and Enes go for a walk in the forest but they cannot remember the way back home. They stand in the middle of the forest where there are six crossroads. Since their cell phones batteries have run out, they cannot contact anyone for help. They begin to jump in place because it is very cold. While Osman jumps, something falls out of his pocket. When they lean down to see what it is, they realize that it is some kind of dice. He says, "We can decide which road to take according to the number that we roll using the dice." Since they have no other choice, they agree to this idea. The number they roll is 4 and therefore, the three friends agree to take the fourth road. After a while, they see the lights of the city. Erdem wants to call his parents immediately. However, the battery in Erdem's cell phone has run out of power and he cannot remember the phone number of his parents. Yet he goes to the first telephone booth he finds and tries to remember. He starts to think, "My dad's phone number definitely begins with 0-5 and there are no symbols like # or *."

a) What is the probability of Erdem's father's phone number beginning with 0-5? How else can we state this situation in a digital manner? (expressing numbers in different types)

b) How would you refer to this situation if you think about it as a type of probability event in general? (describing a context problem in mathematical terms)

c) What is the probability of the number of Erdem's father having the # and * symbols in his cell phone number? How else can we state this situation in a digital manner? (expressing numbers in different types)

d) What would you call this situation if you think about it as a type of probability event in general? (describing a context problem in mathematical terms)

When Erdem cannot remember his father's cell number, he calls his grandmother instead and informs her about the situation. His grandmother calls Erdem's parents and tells them not to worry. Erdem's parents are happy about the news and decide to celebrate. They invite the families of Osman and Enes to the celebration. Erdem's mother prepares a surprise game. She makes them draw one marble from a plastic bag that is not transparent and which holds six red and three white marbles. Erdem, Osman and Enes will draw marbles one by one and never return the marbles to the plastic bag again. Whoever draws the white marble first will receive strawberry cake as a reward. However, Enes, who will draw the last, objects to this situation and tells them that it is not fair.

e) Why, in your opinion, does Enes object? What do you recommend be done to ensure a fair draw? (solving a problem situated in daily life)

Note: In Turkey, cell phone numbers always begin with the numbers 05 and never begins with symbols such as # or *.

For this study, the statement "(a) acceptable responses" was used instead of "correct responses" in cases where responses to questions in the study are true. Here, the use of the statement "acceptable responses" may be justified by the fact that there is no single true answer for the questions and that some responses may vary according to the student's interpretation. Wrong answers were examined according to two categories. Responses that stemmed from a lack of information were expressed as "(b) wrong responses", while mistakes stemming from misinterpretations or emotional thinking were expressed as "(c) wrong perspective.

RESULTS

In the first step of the research, horizontal mathematization processes were identified and classified, and the possible reasons for these misconceptions were estimated. Horizontal mathematization processes were examined as three different elements in this study:

1. Describing mathematical terms in a context problem: The story provides in the study includes the definition of a ‘certain event’ and ‘impossible event’ about probability subject.
2. Expressing numbers in different types: the story includes an expression of possibility in the form of numerically different forms.
3. Solving a problem situated in daily life: the story expresses the discovery of a solution and making suggestions for an event that includes everyday life events to indicate probability.

Firstly, the examinations conducted for the “describing mathematical terms in a context problem” item, which was aimed at the mathematical terms in the story, were expressed separately as a certain and impossible event. Here, according to the context of the story, it was expected that the definite existence of the numbers at the beginning of the cell phone numbers would be expressed as the certain event and impossible event.

Table 1. The findings regarding Describing Mathematical Terms

Type of Responses	Describing Mathematical Terms	
	Definition Of Certain Event (f)	Definition Of Impossible Event (f)
Acceptable Responses	18	19
Wrong Responses	15	12
Wrong Perspective	2	4

In responses defined as acceptable responses, some students answered by using only the concepts certain or impossible, whereas they were expected to write certain event or impossible event. Some students replied using certain possibility or impossible possibility instead of event. These replies were also accepted as being correct. In responses that were accepted as wrong perspective, although the students knew the meaning of the certain event concept, they did not answer by providing proper definitions and used misconceptions like whole possibility or being possibility. Similarly, while they expressed the impossible event concept, they used expressions like “there is no such possibility” or “non-being possibility”.

For the second item, examinations were made for the item “expressing numbers in different types”. This concerned expressing the definite and impossible event concepts in numerically different ways. For this purpose, it was expected that digital representations that express 1 would be given as an answer for the definite event in the story and 0 be given as an answer for impossible events.

Table 2. The findings regarding Expressing Numbers in Different Types

Type of Responses	Expressing Numbers in Different Types	
	Numerical Expression Of Certain Event (f)	Numerical Expression Of Impossible Event (f)
Acceptable Responses	20	21
Wrong Responses	13	9
Wrong Perspective	2	5

It is possible to argue that the students were more successful at expressing concepts regarding the numerical probability in the impossible event. Although some students were unable to express the certain event and impossible event concepts in the probability subject, they were able to express them in a numerical manner. The student encoded as S29 defined the certain event concept in a digital manner as 100% and 12/12 when defining it as a theoretical event. S20 could not express the impossible event concept; however, the student expressed it in a numerical manner as 0/100. S1, S16 and S9 could not express the impossible event concept; however, they stated it digitally as 0. S6 and S11 did not express the impossible and certain event concepts; however, they expressed these concepts correctly in a numerical manner. It was determined that there were

misconceptions in the numerical expressions of the students. Although S27 expressed the certain event concept in a correct manner, s/he expressed it both as 100% and 2/11. Similarly, S25 used the 100% and 1/10 expressions to express the definite event in a numerical manner. S16, S23, S26 and S31 expressed the impossible event concept as 0/0. Similarly, S32 used the $0/\infty$ together with 0. Here, the issue of whether the students knew the undefined and uncertainty concepts may be questioned.

As a final item, the responses given to the context problem were analysed to examine the “solving a problem situated in daily life” concept. In this story, the three friends are asked to draw marbles from a bag in which there were six red marbles and three white marbles, and were told never to put them back in the bag once drawn. Those who drew a white marble would be given a piece of cake. It was expressed that Enes, who is one of the story’s characters, will draws last. Enes was asked why he objected to this draw. The students were also asked to make suggestions for a fair draw.

Table 3. The findings regarding Solving A Problem

Type of Responses	Solving A Problem Situated (f)
Acceptable Responses	24
Wrong Responses	5
Wrong Perspective	6

It was observed that the rate of wrong answers given by students to this question was lower. Since it was a question based on interpretation, most of the students provided acceptable responses. According to the responses given by the students for ensuring a fair draw, the most-frequently made suggestions were “the draw should be made with the condition of putting the marbles back in the bag, give the same plastic bag to all three friends and having them draw marbles at the same time”. The most surprising response was made by the student encoded as s20, who suggested throwing dice and/or tossing a coin instead of drawing marbles from a bag. Five students provided wrong answers to this question. S25 suggested giving a reward to the person who drew the red marble in order to ensure a fair draw. The students encoded as S1, S26 and S29 suggested equalizing the numbers of white and red marbles in order to make the draw fair. The student encoded as S7 suggested increasing the number of white marbles. It can be argued that these students did not consider the order of the draw.

CONCLUSION

In this horizontal mathematization processes were examined according to three main types “describing a context problem in mathematical terms, expressing numbers in different types and solving a problem situated in daily life”. The responses given by students to the three different themes were then analysed. Here, it is possible to claim that the GME approach served as a factor in the context of “the reality principle”. According to the level principle, one of the GME approach principles, the question given that were related to the theme of “solving a problem situated in daily life” is linked to informal learning, and can be answered by any student via an expression of their ideas, and is a question that is based on interpretation. In this instance, students were expected to make suggestions for ensuring a fair draw; this enabled students who did not have any knowledge about probability to answer the question by brainstorming.

The step in this activity that required the most formal knowledge was the “describing a mathematical term” step. Students who were unable to provide correct answers to this question using 1, 100% and 12/12 as digital expressions of the definite event, and using 0, 0%, 0/3 and similar expressions to infer the numerical expression of the impossible event, indicated that even when they did not know the correct definitions for concepts included in this topic, they still had knowledge about the contents of possibility. When the correct answer rates for the students were examined from another perspective, it was found that the least correct answers were given to the questions for the mathematical terms theme, while the most correct answers were given to problems in the solving a problem situated in daily life theme. This indicates that the students were

more successful at subjects based on interpreting situations related to daily life, rather than definitions of clearly mathematical activities. In fact, the main purpose of the RME approach is to support informal learning as opposed to formal learning. In this context, the present study concludes that preparing activities that will help students face everyday problematic situations may be influential for developing positive attitudes towards mathematics by ensuring active participation in the mathematic classes. As Treffers (1978) states, the problem has to be first transformed using an empirical approach of observation, experimentation and inductive reasoning before using strictly mathematical methods in the horizontal mathematization process. By doing so, the understanding of students regarding mathematical concepts and expressions will become easier. Additionally, upper level mathematical skills can be analysed and interpreted by examining vertical mathematization processes that are included in the realistic mathematical education approach.

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